"Risk" in an untrusted setting

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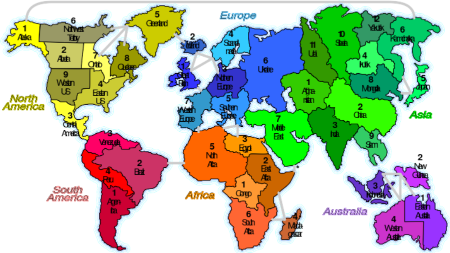
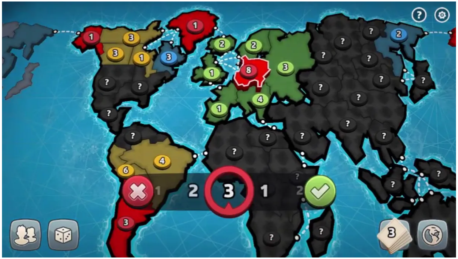
Risk

- *Risk* is a popular strategy board game.
- It is played on a single board, depicting a world map, partitioned into regions.
- A player owns a region of the map by stationing troops within the region.
- Players fight for regions by gambling some of their troops against the troops in the other player's region.



Risk

- *Risk* has a variant called ”fog of war”.
- In this variant, players can only see the number of troops stationed within regions they neighbour.
- This variant is therefore only played online, in a **trusted setup**.



Proposition

- Play fog-of-war Risk in an untrusted setup.
- In the untrusted setup, the same guarantees should be made as the trusted setup, but on a peer-to-peer network.

Rationale

- **Decentralised**
  - Longer lifespans than centralised platforms.
  - More resistant to censorship and can help promote anonymity and privacy.
  - Encourages user freedom.
- **Security**
  - Constantly looking for ways to secure against threats specific to federated and decentralised infrastructures.
  - Security issues can be devastating even to decentralised infrastructures.

State of the art

- Private key encryption.
- Signatures.
- Additive homomorphic encryption.
- **Monero, Zcash**. Decentralised ledgers respectively using the *Bulletproof* and *ZK-SNARK* zero-knowledge proof systems.
- **Web platform**.

Results

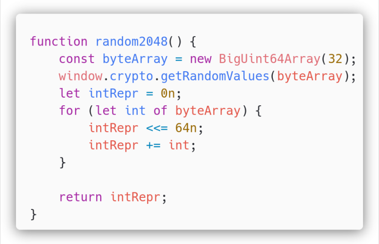
Emulated P2P environment using WebSockets.

Results

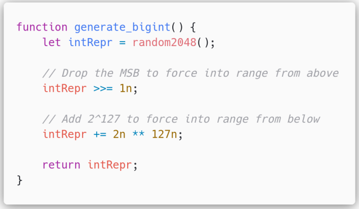
Produce shared random values without beacons using commitment schemes.

Results

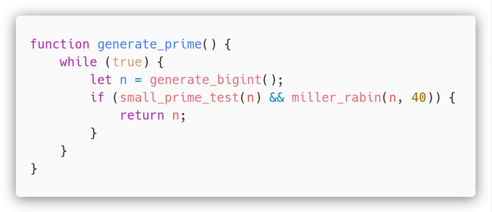
Generating large primes using ECMAScript `BigInt` and Rabin-Miller.



```
function random2048() {
    const byteArray = new BigUint64Array(32);
    window.crypto.getRandomValues(byteArray);
    let intRepr = 0n;
    for (let int of byteArray) {
        intRepr <<= 64n;
        intRepr += int;
    }

    return intRepr;
}
```



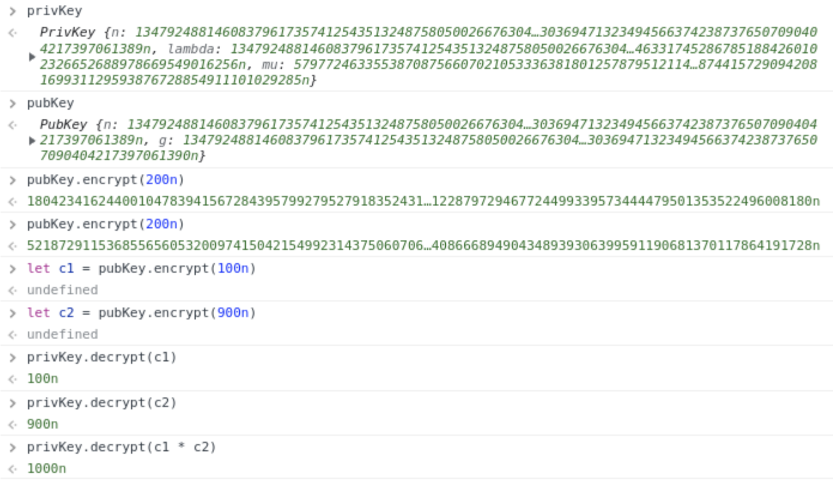
```
function generate_bigint() {
    let intRepr = random2048();

    // Drop the MSB to force into range from above
    intRepr >>= 1n;

    // Add 2^127 to force into range from below
    intRepr += 2n ** 127n;

    return intRepr;
}
```



```
function generate_prime() {
    while (true) {
        let n = generate_bigint();
        if (small_prime_test(n) && miller_rabin(n, 40)) {
            return n;
        }
    }
}
```

Results

Implementation of the Paillier additive homomorphic cryptosystem.



```
> privKey
<·  PrivKey {n: 13479248814608379617357412543513248758050026676304…30369471323494566374238737650709040
 ▶ 4217397061389n, lambda: 13479248814608379617357412543513248758050026676304…46331745286785188426010
   23266526889786695490l6256n, mu: 57977246335538708756607021053336381801257879512114…874415729094208
   16993112959387672885491110l029285n}
> pubKey
<·  PubKey {n: 13479248814608379617357412543513248758050026676304…303694713234945663742387376507090404
 ▶ 217397061389n, g: 13479248814608379617357412543513248758050026676304…303694713234945663742387376 50
   70904042l7397061390n}
> pubKey.encrypt(200n)
<· 18042341624400104783941567284395799279527918352431…12287972946772449933957344447950135352249600818 0n
> pubKey.encrypt(200n)
<· 52187291153685565605320097415042154992314375060706…40866689490434893930639959119068137011786419172 8n
> let c1 = pubKey.encrypt(100n)
<· undefined
> let c2 = pubKey.encrypt(900n)
<· undefined
> privKey.decrypt(c1)
<· 100n
> privKey.decrypt(c2)
<· 900n
> privKey.decrypt(c1 * c2)
<· 1000n
```

Results

Implementation of Risk.

Citations

Image Risk game board by CMG Lee, the asterisk denoting the missing link in the 40th Anniversary Collector's Edition, based on shapes from http://commons.wikimedia.org/wiki/File:Risk_board.svg. 11 November 2008. CC-BY-SA 4.0